Mining & Learning on Graphs

Network Analysis

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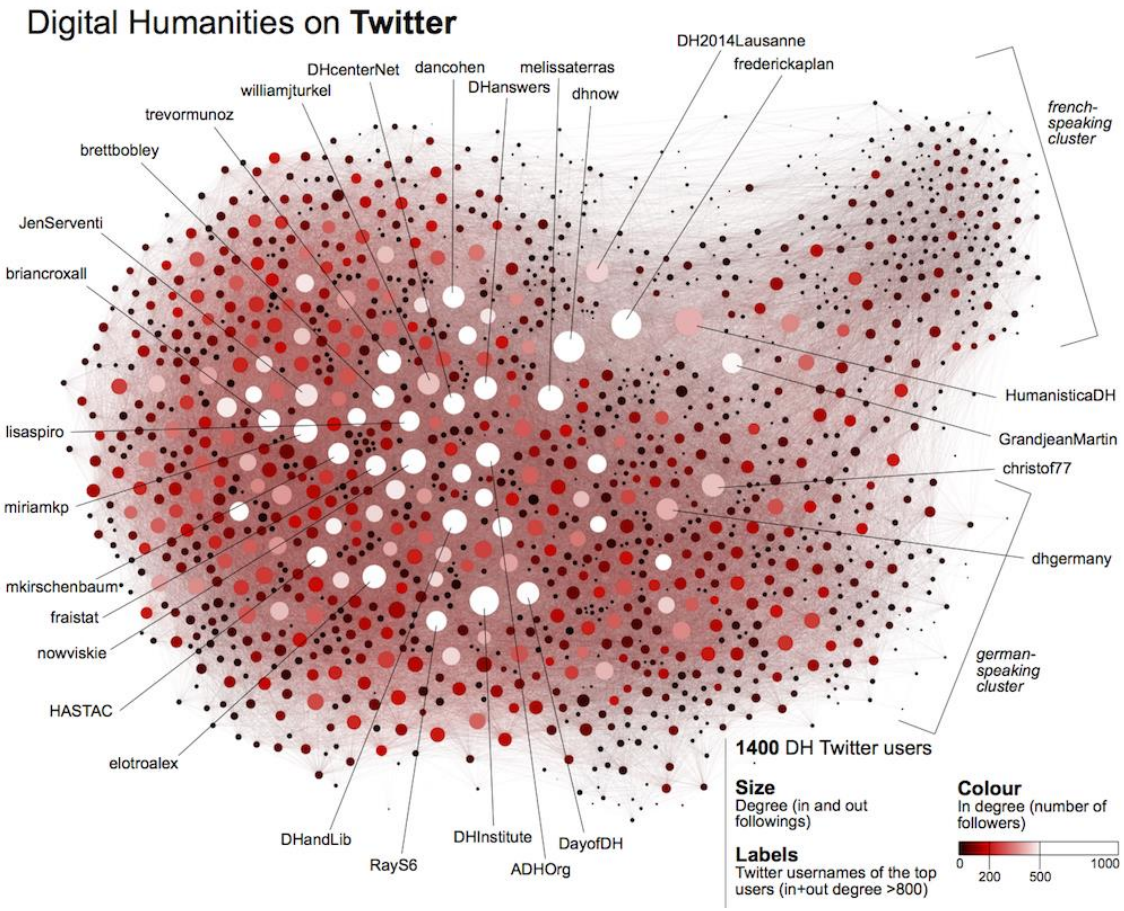
Network Analysis - Motivation

Why network analysis?



Digital Humanities practitioners connect and interact on Twitter

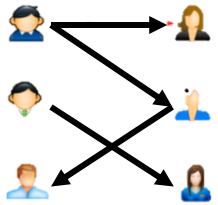
Key influencers

Community

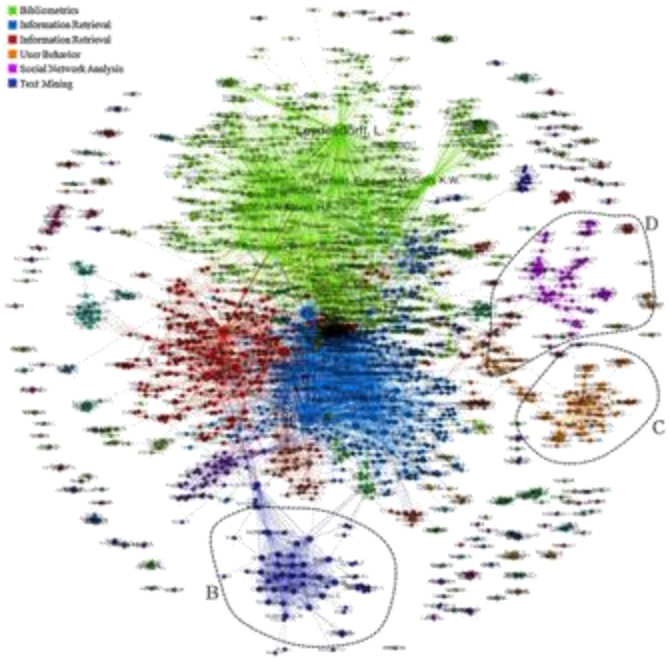
Language-based Cluster

Why network analysis?

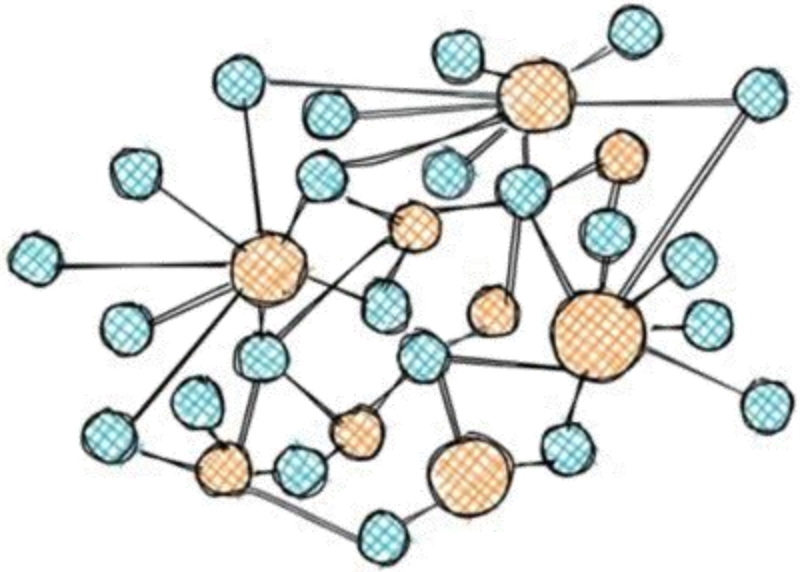
Birds of a feather flock together



Dating Network



Homophily



Heterophily

Network Analysis - Motivation

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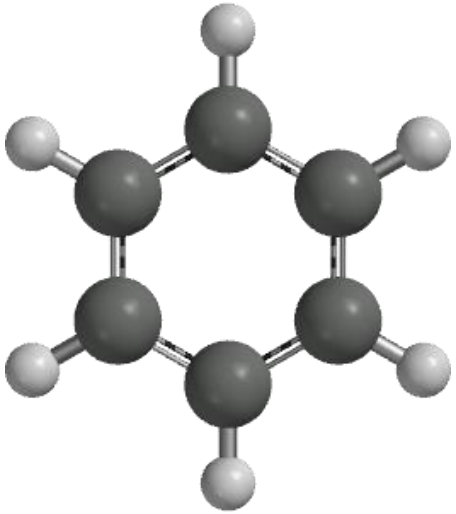
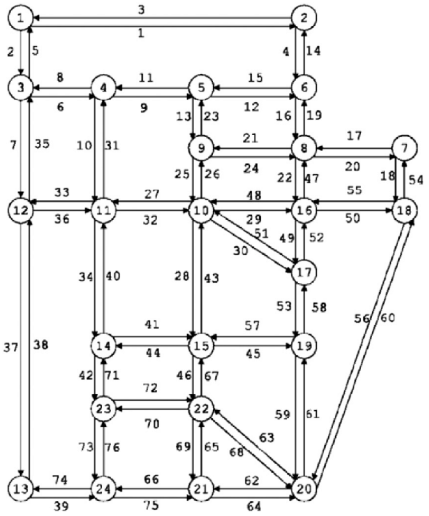


Benzene:
6-cycle



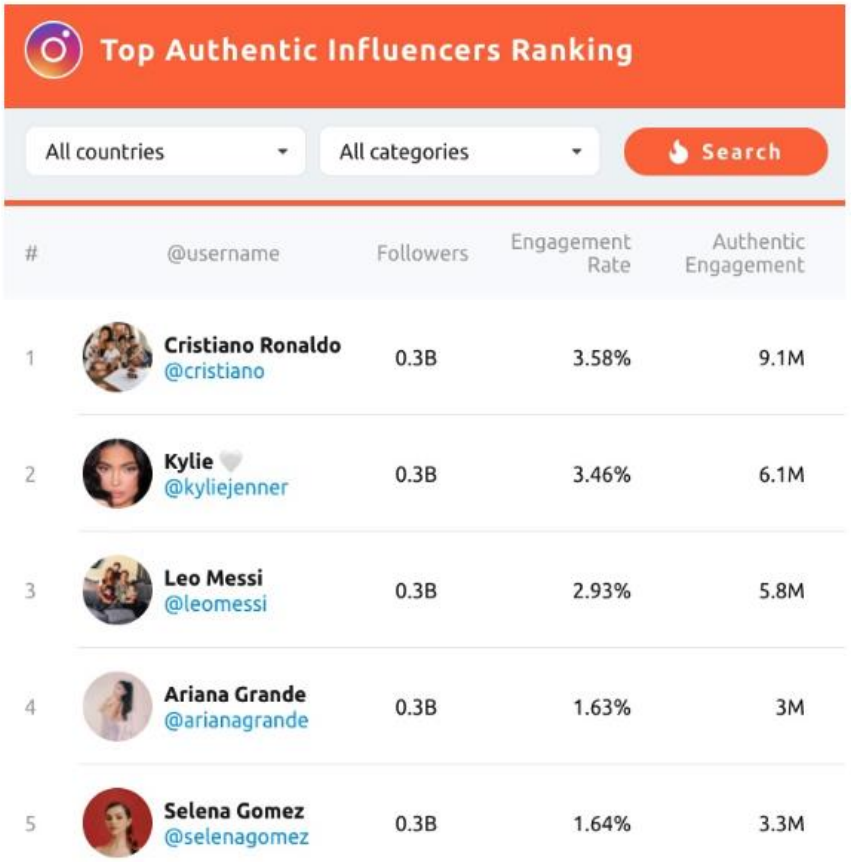
Road:
4-cycle

Network Centrality

- Which nodes in the graph are “important”?

Motivation

- Which users in a social network are more influential?

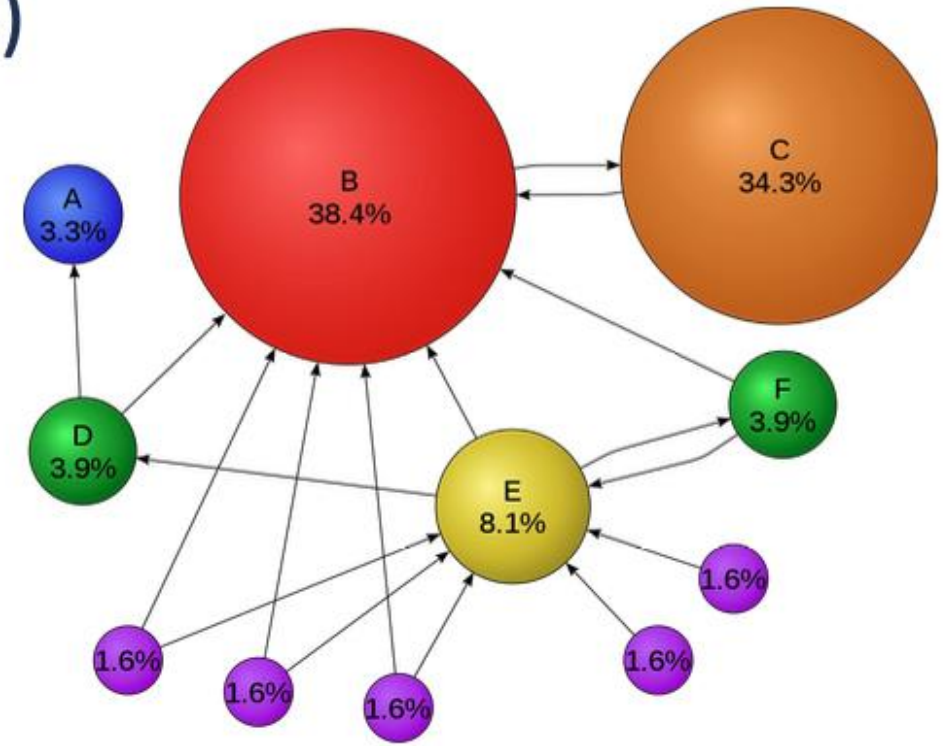


Note: Influence maximization will be a dedicated topic later

Motivation

- Which employees are the most central to a company?
- Which web pages are more likely of interest/important to show you?

Is a measure that ranks the nodes of a graph to answer the mentioned types of questions (and more!)



Note that today we will focus on undirected graphs

Network Centrality

- Basic measures of centrality/prestige
  - Mostly from a social perspective
- Path-based
  - Closeness, Betweenness, Katz
- Eigenvector-based
  - Eigenvector, Katz, PageRank
- Others
  - Hubs and Authorities, Goodness and Fairness

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Network Centrality – Sociology Perspective

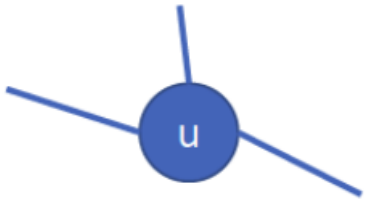
- For a social network let us consider the graph of "actors" (i.e., nodes) and then defines "importance" measures accordingly

- Actor prestige
  - Motivation: The more ties an actor has the higher their importance
  - Note in a directed network would this be incoming or outgoing?
- Actor centrality
  - Motivation: Involvement with other actors, how many ties, who are they linked with, etc.

Undirected Degree Centrality

- Actor prestige (i.e., Degree Centrality)
  - Motivation: The more ties an actor has the higher their importance

$C_D(v) = 6$



$C_D(u) = 3$

Directed Degree Centrality

- Actor prestige (i.e., Degree Centrality)
  - Motivation: The more ties an actor has the higher their importance
  - Note in directed networks we use incoming links (and not outgoing)

$C_D(v) = 1$

$C_D(u) = 2$

E.g., Twitter

- Why we might prefer using incoming (i.e., "Followers") here, but this still isn't taking full information into account when deciding a ranking

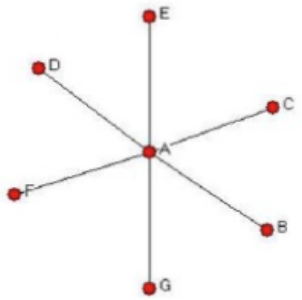
Problems: Degree Centrality

- Undirected Degree Centrality: number of neighbors 1-hop away from the node

$$C_D(i) = k_i = \sum_j \mathbf{A}_{ij} = \sum_j \mathbf{A}_{ji}$$

How to compare with other networks?

$$C_{D^*}(i) = \frac{1}{n-1} C_D(i)$$



Problems: Degree Centrality

$$C_D(i) = k_i = \sum_j \mathbf{A}_{ij} = \sum_j \mathbf{A}_{ij}$$

$$C_{D^*}(i) = \frac{1}{n-1} C_D(i)$$

Which users have high centrality?

Those with direct contact to many others

Which users have low centrality?

Those not very active or peripheral users

Network Centrality

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  - Eigenvector, Katz, PageRank
- Others
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Path-based Centrality

- Built on the idea that the centrality of a node is in relation to how they are connected with others in the network

- Closeness Centrality:
  - How close an actor is to all other actors in the network

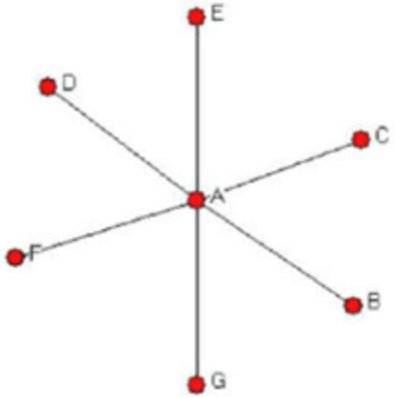
- Betweenness Centrality:
  - The # of shortest paths that flow through that actor

Closeness Centrality

Closeness Centrality: Alex Bavelas, 1948

- How close an actor is to all other actors in the network

$$C_C(i) = \frac{1}{\sum_j d(i,j)}$$



Problem: How to compare across graphs?

Normalized Closeness Centrality

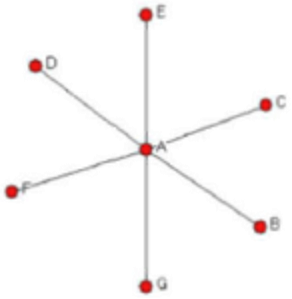
Closeness Centrality:

- How close an actor is to all other actors in the network

$$C_C(i) = \frac{1}{\sum_j d(i,j)}$$

Normalized Closeness Centrality:

$$C_{C^*}(i) = (n-1)C_C(i)$$



Problem: What if graph is disconnected?

Harmonic Centrality

Closeness Centrality:

$$C_C(i) = \frac{1}{\sum_j d(i,j)}$$

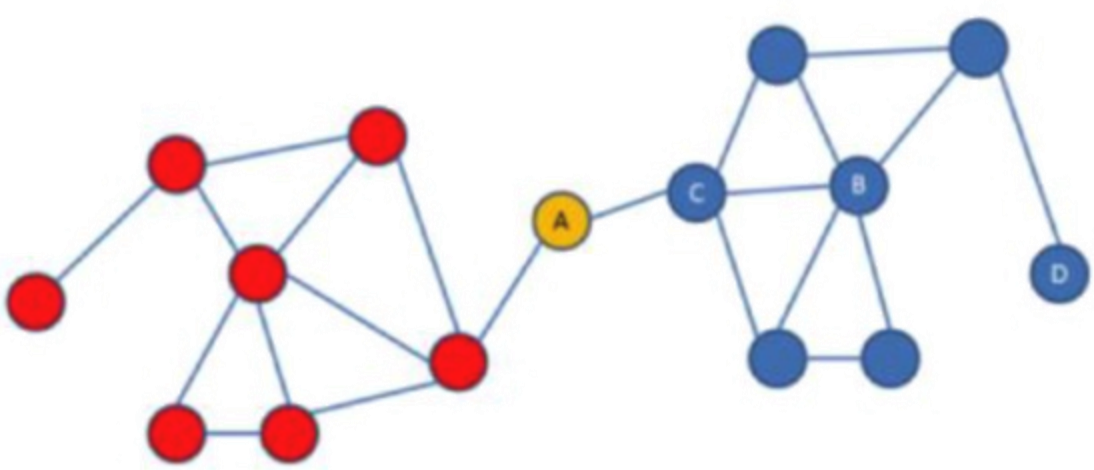
Harmonic Centrality:

Take the harmonic mean by inverting all distances

$$C_H(i) = \sum_j \frac{1}{d(i,j)}$$

Betweenness Centrality

- Betweenness Centrality: Linton Freeman, 1977
  - The # of shortest paths σ that flow through that actor



Betweenness Centrality

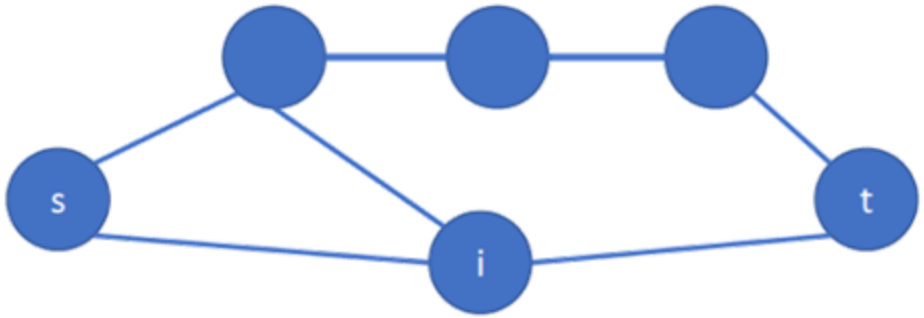
- **Betweenness Centrality:** Linton Freeman, 1977
  - The # of shortest paths σ that flow through that actor

$$C_B(i) = \sum_{s \neq t \neq i} \frac{\sigma_{st}(i)}{\sigma_{st}}$$

where

$\sigma_{st}(i)$ number of shortest paths through node I

σ_{st} number of shortest paths



$\sigma_{st}(i) = 1$

$\sigma_{st} = 1$

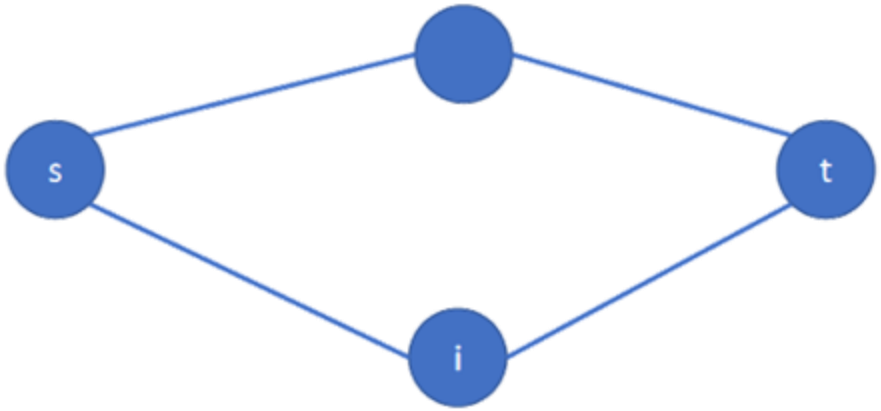
Betweenness Centrality

- Betweenness Centrality: Linton Freeman, 1977
  - The # of shortest paths σ that flow through that actor

$$C_B(i) = \sum_{s \neq t \neq i} \frac{\sigma_{st}(i)}{\sigma_{st}}$$

$\sigma_{st}(i) = 1$

$\sigma_{st} = 2$



Betweenness Centrality

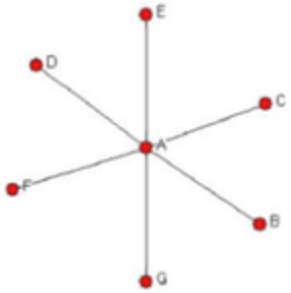
- **Betweenness Centrality:** Linton Freeman, 1977
  - The # of shortest paths σ that flow through that actor

$$C_B(i) = \sum_{s \neq t \neq i} \frac{\sigma_{st}(i)}{\sigma_{st}}$$

- **Normalized Betweenness Centrality:**

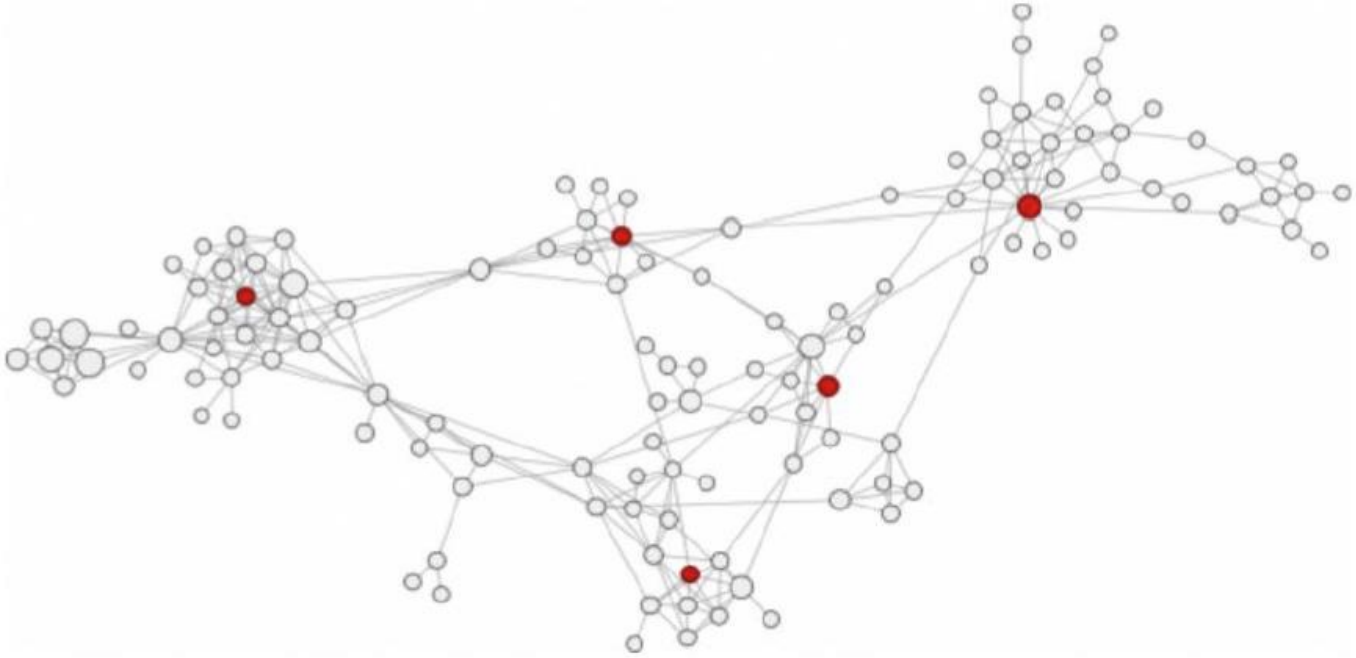
$$C_{B^*}(i) = \frac{2}{(n-1)(n-2)} C_B(i)$$

How did we get this normalization?



Recap early path-based

- Closeness Centrality



Recap early path-based

- Betweenness Centrality

Katz Centrality

Leo Katz, 1953

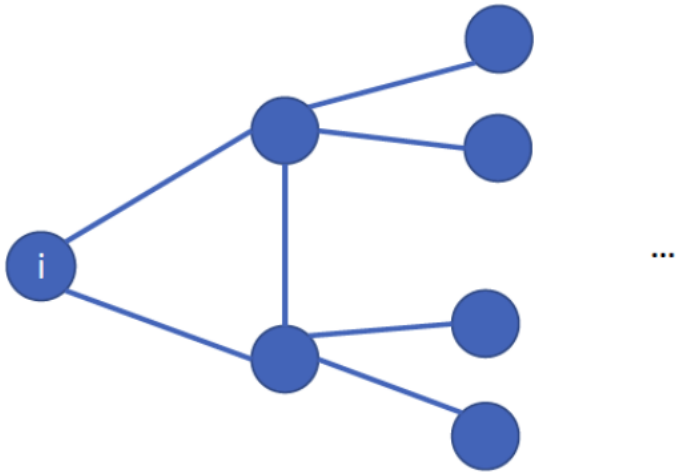
- Degree Centrality was based on counting neighbors
- What if we want to consider how many neighbors' neighbors, etc.

- Weighted count of all paths connecting to that node with a discount on longer paths
  - Weight of path of length n is β^n
  - [Note that $\beta < \frac{1}{\lambda_1}$ but we will not discuss this]

$$c_i = \beta \sum_j \mathbf{A}_{ij} + \beta^2 \sum_j \mathbf{A}_{ij}^2 + \beta^3 \sum_j \mathbf{A}_{ij}^3 + \ \ldots$$

Katz Centrality

$$c_i = \beta \sum_j \mathbf{A}_{ij} + \beta^2 \sum_j \mathbf{A}_{ij}^2 + \beta^3 \sum_j \mathbf{A}_{ij}^3 + \ \ldots$$

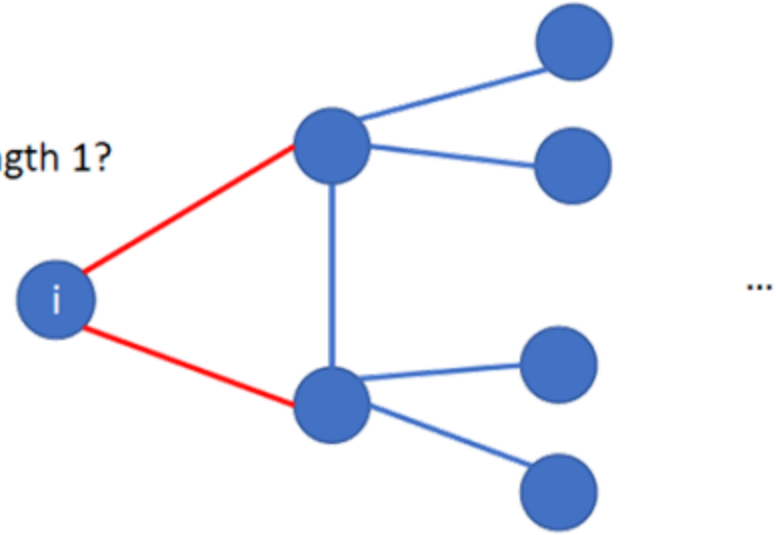


Katz Centrality

$$c_i = \beta \sum_j \mathbf{A}_{ij} + \beta^2 \sum_j \mathbf{A}^2_{ij} + \beta^3 \sum_j \mathbf{A}^3_{ij} + \ \ldots$$

How many paths of length 1?

2



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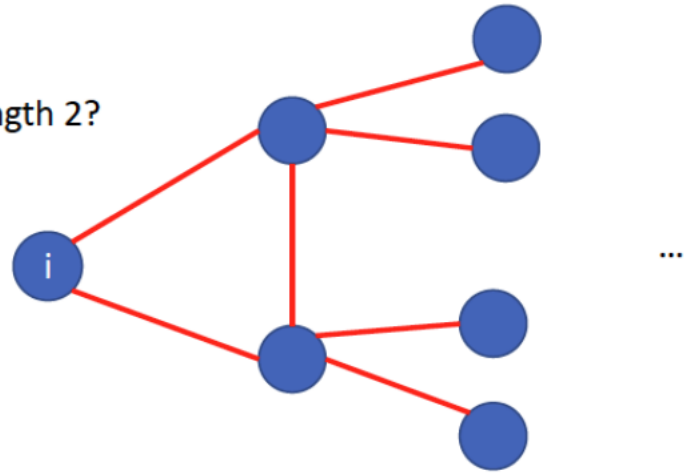
Katz Centrality

$$c_i = \beta \sum_j \mathbf{A}_{ij} + \beta^2 \sum_j \mathbf{A}_{ij}^2 + \beta^3 \sum_j \mathbf{A}_{ij}^3 + \ \ldots$$

How many paths of length 2?



...

Katz Centrality

$$c_i = 1 + \beta \sum_j \mathbf{A}_{ij} + \beta^2 \sum_j \mathbf{A}^2_{ij} + \beta^3 \sum_j \mathbf{A}^3_{ij} + \ \ldots$$

$$\mathbf{c} = (\mathbf{I} + \beta\mathbf{A} + \beta^2\mathbf{A}^2 + \beta^3\mathbf{A}^3 + \ \ldots)\mathbf{1}$$

$$\mathbf{c} = \sum_{n}^{\infty} (\beta^n \mathbf{A}^n)\mathbf{1}$$

$$\mathbf{c} = (\mathbf{I} - \beta A)^{-1}\mathbf{1}$$

Intuition:
Taylor series expansion

$$\frac{1}{1-x} \approx 1 + x + x^2 + \cdots$$

Modified Katz

- Newman 2010

$$\mathbf{c}_i = \alpha \sum_j \mathbf{A}_{ij} \boldsymbol{c}_j + \beta_i$$

$$c = (\mathbf{I} - \alpha \mathbf{A})^{-1} \beta$$

- Bonacich 2001 (Alpha-Centrality)

$$\mathbf{c}_i = \alpha \sum_j \mathbf{A}_{ij} \boldsymbol{c}_j + 1$$

$$c = (\mathbf{I} - \alpha \mathbf{A})^{-1} \mathbf{1}$$

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Any Question?